

H/W. 9.

Perform a two-way classification to the following data.

| Varieties | Block A | Block B | Block C | Block D |
|-----------|---------|---------|---------|---------|
| I         | 8       | 5       | 5       | 7       |
| II        | 7       | 6       | 4       | 4       |
| III       | 3       | 6       | 5       | 4       |

Sol:-

|       | $x_1$           | $x_2$           | $x_3$           | $x_4$           | Total           | $x_1^2$            | $x_2^2$           | $x_3^2$           | $x_4^2$           |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|--------------------|-------------------|-------------------|-------------------|
| $y_1$ | 8               | 5               | 5               | 7               | $\sum y_1 = 25$ | 64                 | 25                | 25                | 49                |
| $y_2$ | 7               | 6               | 4               | 4               | $\sum y_2 = 21$ | 49                 | 36                | 16                | 16                |
| $y_3$ | 3               | 6               | 5               | 4               | $\sum y_3 = 18$ | 9                  | 36                | 25                | 16                |
| Total | $\sum x_1 = 18$ | $\sum x_2 = 17$ | $\sum x_3 = 14$ | $\sum x_4 = 15$ | $T = 64$        | $\sum x_1^2 = 122$ | $\sum x_2^2 = 97$ | $\sum x_3^2 = 66$ | $\sum x_4^2 = 81$ |

Step 1:

$H_0$  - There's no significant difference b/w the column means or row means.

$H_1$  - There is significant difference b/w the column means or row means.

Step 2:  $N = 12$ .

Step 3:  $T = 64$ .

Step 4:  $\frac{T^2}{N} = \frac{64^2}{12} = 341.33$ .

Step 5:

$$TSS = \sum x_1^2 + \sum x_2^2 + \sum x_3^2 + \sum x_4^2 - T^2/N$$

$$= 122 + 97 + 66 + 81 - 341.33$$

$$TSS = 24.67$$

step 6:

$$SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{18^2}{3} + \frac{17^2}{3} + \frac{14^2}{3} + \frac{15^2}{3} - 341.33$$

$$= 18^2/3 + 17^2/3 + 14^2/3 + 15^2/3 = 108 + 96.33 + 65.33 + 75 - 341.33$$

$$= 108 + 96.33 + 65.33 + 75 - 341.33$$

$$SSC = 3.33 = -82.83$$

step 7:

$$SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} - \frac{T^2}{N}$$

$$= 156.25 + 110.25 + 81 - 341.33$$

$$SSR = 6.17$$

step 8:

$$SSE = TSS - SSC - SSR$$

$$= 26.67 - 3.33 - 6.17$$

$$SSE = 15.17$$

step 9: ANOVA Table:

| Source of Variation | Sum of Squares | DoF                          | Mean Squares   | Variance ratio   | Table Value                    |
|---------------------|----------------|------------------------------|--|--|--------------------------------|
| Between Column      | SSC = 3.33     | c-1<br>= 4-1<br>= 3          | MSC = $\frac{SSC}{c-1}$<br>= 3.33/3<br>MSC = 1.11                    | $F_c = \frac{MSE}{MSC}$<br>= $\frac{2.528}{1.11}$<br>F <sub>c</sub> = 2.277  | F <sub>c</sub> (6,3)<br>= 8.94 |
| Between Rows        | SSR = 6.17     | r-1<br>= 3-1<br>= 2          | MSR = $\frac{SSR}{r-1}$<br>= $\frac{6.17}{2}$<br>MSR = 3.085         | $F_r = \frac{MSR}{MSE}$<br>= $\frac{3.085}{2.528}$<br>F <sub>r</sub> = 1.220 | F <sub>r</sub> (2,6)<br>= 5.14 |
| Error               | SSE = 15.17    | (c-1)(r-1)<br>= 3 x 2<br>= 6 | MSE = $\frac{SSE}{(c-1)(r-1)}$<br>= $\frac{15.17}{6}$<br>MSE = 2.528 |  |                                |

step 10: conclusion:

Here  $F_{c\text{ cal}} < F_{c\text{ tab}}$ ,  $F_{r\text{ cal}} < F_{r\text{ tab}}$ .

$\therefore$  In both cases, we accept  $H_0$

A variable trial was conducted on wheat with a vortet

Latin Square Design:-

Latin square Design is very popular in agricultural research where it is not possible to have a large number of (objects) subjects.

Working Rule:-

- step 1: Find N
- step 2: Find T
- step 3: Find  $T^2/N$
- step 4: Find TSS
- step 5: Find SSC
- step 6: Find SSR, Find SSK
- step 7: ANOVA table
- step 8: Conclusion.
- step 9:

| Source of Variation | Sum of Squares | DoF          | Mean Sum of Squares            | Variance ratio          | Table Value |
|---------------------|----------------|--------------|--------------------------------|-------------------------|-------------|
| Between column      | SSC            | k-1          | $MSC = \frac{SSC}{k-1}$        | $F_c = \frac{MSC}{MSE}$ |             |
| Between rows        | SSR            | k-1          | $MSR = \frac{SSR}{k-1}$        | $F_r = \frac{MSR}{MSE}$ |             |
| Between treatments  | SSK            | k-1          | $MSK = \frac{SSK}{k-1}$        | $F_T = \frac{MSK}{MSE}$ |             |
| Error               | SSE            | $(k-1)(k-2)$ | $MSE = \frac{SSE}{(k-1)(k-2)}$ | -                       | -           |

Problems:-

10. The following is the latin square of the design when four variety of seeds are tested setup the analysis variance table and state your conclusion. You may carry out suitable change in origin and scale.

|   |     |   |     |   |     |   |     |
|---|-----|---|-----|---|-----|---|-----|
| A | 105 | B | 95  | C | 125 | D | 115 |
| C | 115 | D | 125 | A | 105 | B | 105 |
| D | 115 | C | 95  | B | 105 | A | 115 |
| B | 95  | A | 135 | D | 95  | C | 115 |

Solution:-

Origin = Subtract 100 then divided by 5.

|       |   | $X_1$ |   | $X_2$ |   | $X_3$ |   | $X_4$ |
|-------|---|-------|---|-------|---|-------|---|-------|
| $Y_1$ | A | 1     | B | -1    | C | 5     | D | 3     |
| $Y_2$ | C | 3     | D | 5     | A | 1     | B | 1     |
| $Y_3$ | D | 3     | C | -1    | B | 1     | A | 3     |
| $Y_4$ | B | -1    | A | 7     | D | -1    | C | 3     |

|       | $X_1$          | $X_2$           | $X_3$          | $X_4$           | Total           | $X_1^2$           | $X_2^2$           | $X_3^2$           | $X_4^2$           |
|-------|----------------|-----------------|----------------|-----------------|-----------------|-------------------|-------------------|-------------------|-------------------|
| $Y_1$ | 1              | -1              | 5              | 3               | $\sum Y_1 = 8$  | 1                 | 1                 | 25                | 9                 |
| $Y_2$ | 3              | 5               | 1              | 1               | $\sum Y_2 = 10$ | 9                 | 25                | 1                 | 1                 |
| $Y_3$ | 3              | -1              | 1              | 3               | $\sum Y_3 = 6$  | 9                 | 1                 | 1                 | 9                 |
| $Y_4$ | -1             | 7               | -1             | 3               | $\sum Y_4 = 8$  | 1                 | 49                | 1                 | 9                 |
| Total | $\sum X_1 = 6$ | $\sum X_2 = 10$ | $\sum X_3 = 6$ | $\sum X_4 = 10$ | $T = 32$        | $\sum X_1^2 = 20$ | $\sum X_2^2 = 76$ | $\sum X_3^2 = 28$ | $\sum X_4^2 = 28$ |

$H_0$ : There's no significant difference b/w column means, row means and treatments.  
 $H_1$ : There's significant difference b/w column means, row means and treatments.

Step 1:  $N = 16$ .

Step 2:  $T = 32$

Step 3:  $\frac{T^2}{N} = \frac{32^2}{16} = 64$ .

Step 4:  $TSS = \sum x_1^2 + \sum x_2^2 + \sum x_3^2 + \sum x_4^2 - \frac{T^2}{N}$   
 $= 20 + 76 + 28 + 28 - 64$

$TSS = 88$

Step 5:  $SSC = \frac{(\sum x_1)^2}{N_1} + \frac{(\sum x_2)^2}{N_1} + \frac{(\sum x_3)^2}{N_1} + \frac{(\sum x_4)^2}{N_1} - \frac{T^2}{N}$   
 $= \frac{6^2}{4} + \frac{10^2}{4} + \frac{6^2}{4} + \frac{10^2}{4} - 64$

$SSC = 4$

Step 6:  $SSR = \frac{(\sum y_1)^2}{N_2} + \frac{(\sum y_2)^2}{N_2} + \frac{(\sum y_3)^2}{N_2} + \frac{(\sum y_4)^2}{N_2} - \frac{T^2}{N}$   
 $= \frac{8^2}{4} + \frac{10^2}{4} + \frac{6^2}{4} + \frac{8^2}{4} - 64$

$SSR = 2$ .

Step 7: SSK: Arrange the elements on the order of treatments.

To find SSK:-

|   |    |   |    |    | Total |
|---|----|---|----|----|-------|
| A | 1  | 1 | 3  | 7  | 12    |
| B | -1 | 1 | 1  | -1 | 0     |
| C | 5  | 3 | -1 | 3  | 10    |
| D | 3  | 5 | 3  | -1 | 10    |

$SSK = \frac{12^2}{4} + \frac{0^2}{4} + \frac{10^2}{4} + \frac{10^2}{4} - \frac{T^2}{N} [64]$

$SSK = 22$

Step 8:

$SSE = TSS - SSC - SSR - SSK$

$= 88 - 4 - 2 - 22$

$SSE = 60$

Step 9: ANOVA Table:-

| Source of Variation | Sum of Squares | DoF                      | Mean Squares                                       | Variance ratio                                   | Table Value       |
|---------------------|----------------|--------------------------|--|--|-------------------|
| Between columns     | $SSC = 4$      | $k-1 = 4-1 = 3$          | $MSC = \frac{SSC}{k-1} = \frac{4}{3} = 1.33$       | $F_c = \frac{MSE}{MSC} = \frac{10}{1.33} = 7.52$ | $F_c(6,3) = 8.94$ |
| Between rows        | $SSR = 2$      | $k-1 = 3$                | $MSR = \frac{SSR}{k-1} = \frac{2}{3} = 0.69$       | $F_R = \frac{MSE}{MSR} = \frac{10}{0.69} = 14.9$ | $F_R(6,3) = 8.94$ |
| Between treatments  | $SSK = 22$     | $k-1 = 3$                | $MSK = \frac{SSK}{k-1} = \frac{22}{3} = 7.33$      | $F_T = \frac{MSE}{MSR} = \frac{10}{7.33} = 1.36$ | $F_T(6,3) = 8.94$ |
| Errors              | $SSE = 60$     | $(k-1) \times (k-2) = 6$ | $MSE = \frac{SSE}{(k-1)(k-2)} = \frac{60}{6} = 10$ |  |                   |

Step 10: Conclusion:-

cal  $F_c < Tab F_c$  ; So we accept  $H_0$ .

cal  $F_R > Tab F_R$  ; So we reject  $H_0$ .

cal  $F_T < cal F_T$  ; So we accept  $H_0$ .

A variable trial was conducted on wheat with 4 varieties in a Latin square design. The plan of the experiment and the yield are given below. Analyse the data and interpret the result.

|   |    |   |    |   |    |   |    |
|---|----|---|----|---|----|---|----|
| C | 25 | B | 23 | A | 20 | D | 20 |
| A | 19 | D | 19 | C | 21 | B | 18 |
| B | 19 | A | 14 | D | 17 | C | 20 |
| D | 17 | C | 20 | B | 21 | A | 15 |

Sol:- Origin = 20.

|   |    |   |    |   |    |   |    |
|---|----|---|----|---|----|---|----|
| C | 5  | B | 3  | A | 0  | D | 0  |
| A | -1 | D | -1 | C | 1  | B | -2 |
| B | -1 | A | -6 | D | -3 | C | 0  |
| D | -3 | C | 0  | B | 1  | A | -5 |

|       | $X_1$          | $X_2$           | $X_3$           | $X_4$           | Total            | $X_1^2$           | $X_2^2$           | $X_3^2$           | $X_4^2$           |
|-------|----------------|-----------------|-----------------|-----------------|------------------|-------------------|-------------------|-------------------|-------------------|
| $Y_1$ | 5              | 3               | 0               | 0               | $\sum y_1 = 8$   | 25                | 9                 | 0                 | 0                 |
| $Y_2$ | -1             | -1              | 1               | -2              | $\sum y_2 = -3$  | 1                 | 1                 | 1                 | 4                 |
| $Y_3$ | -1             | -6              | -3              | 0               | $\sum y_3 = -10$ | 1                 | 36                | 9                 | 0                 |
| $Y_4$ | -3             | 0               | 1               | -5              | $\sum y_4 = -7$  | 9                 | 0                 | 1                 | 25                |
| Total | $\sum X_1 = 0$ | $\sum X_2 = -4$ | $\sum X_3 = -1$ | $\sum X_4 = -7$ | $T = -12$        | $\sum X_1^2 = 36$ | $\sum X_2^2 = 46$ | $\sum X_3^2 = 11$ | $\sum X_4^2 = 29$ |

Step 1:  $N = 16$

$H_0$ : There's no significant difference b/w column means, row means and treatments.

Step 2:  $T = -12$

Step 3:  $T^2/N = (-12)^2/16 = 9$   $H_1$ : There is significant

Step 4:  $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$  difference b/w column means, row means and treatments.  
 $= 36 + 46 + 11 + 29 - 9$   
 $TSS = 113$

$$\text{Step 5: } SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{0^2}{4} + \frac{(-4)^2}{4} + \frac{(-1)^2}{4} + \frac{(-7)^2}{4} - 9$$

$$SSC = 7.5$$

$$\text{Step 6: } SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{8^2}{4} + \frac{(-3)^2}{4} + \frac{(-10)^2}{4} + \frac{(-7)^2}{4} - 9$$

$$SSR = 46.5$$

Step 7: To find SSK.

|   | Total |    |    |    |     |
|---|-------|----|----|----|-----|
| A | 0     | -1 | -6 | -5 | -12 |
| B | 3     | -2 | -1 | 1  | 1   |
| C | 5     | 1  | 0  | 0  | 6   |
| D | 0     | -1 | -3 | -3 | -7  |

$$SSK = \frac{(\sum A)^2}{N_3} + \frac{(\sum B)^2}{N_3} + \frac{(\sum C)^2}{N_3} + \frac{(\sum D)^2}{N_3} - \frac{T^2}{N}$$

$$= \frac{(-12)^2}{4} + \frac{1^2}{4} + \frac{6^2}{4} + \frac{(-7)^2}{4} - 9$$

$$SSK = 48.5$$

Step 8:

$$SSE = TSS - SSC - SSR - SSK$$

$$= 113 - 7.5 - 46.5 - 48.5$$

$$SSE = 10.5 \%$$



### Step 9: ANOVA Table:-

| Source of variation | Sum of squares | Dof                           | Mean of squares  | Variance ratio  | Table Value       |
|---------------------|----------------|-------------------------------|--|---|-------------------|
| Between columns     | SSC = 7.5      | $k-1 = 4-1 = 3$               | $MSC = \frac{SSC}{k-1} = \frac{7.5}{3}$ $MSC = 2.5$    | $F_c = \frac{MSC}{MSE} = \frac{2.5}{1.75}$ $F_c = 1.428$  | $F_c(3,6) = 4.76$ |
| Between rows        | SSR = 46.5     | $k-1 = 4-1 = 3$               | $MSR = \frac{SSR}{k-1} = \frac{46.5}{3}$ $MSR = 15.5$  | $F_R = \frac{MSR}{MSE} = \frac{15.5}{1.75}$ $F_R = 8.857$ | $F_R(3,6) = 4.76$ |
| Between treatments  | SSK = 48.5     | $k-1 = 3$                     | $MSK = \frac{SSK}{k-1} = \frac{48.5}{3}$ $MSK = 16.17$ | $F_T = \frac{MSK}{MSE} = \frac{16.17}{1.75}$ $F_T = 9.24$ | $F_T(3,6) = 4.76$ |
| Error               | SSE = 10.5     | $(k-1)(k-2) = 3 \times 2 = 6$ | $MSE = \frac{SSE}{6} = \frac{10.5}{6}$ $MSE = 1.75$    |   |                   |

### Step 10: Conclusion:-

$F_c \text{ cal} < F_c \text{ Tab}$ , we accept  $H_0$ .

$F_R \text{ cal} > F_R \text{ tab}$ , we reject  $H_0$ .

$F_T \text{ cal} > F_T \text{ Tab}$ , we reject  $H_0$ .

12. Five varieties of wheat, A, B, C, D and E were tried. The gross size of the plot was  $18 \times 22$  feet, the net plot being  $14 \times 18$  feet. Thus the whole experiment occupied an area  $90 \times 110$  ft. The plan the varieties shown in each plot and yields obtained in kg. are given in following table. Carry out an analysis of variance. What inference can you draw from the data given?

|   |     |   |     |   |     |   |     |   |    |
|---|-----|---|-----|---|-----|---|-----|---|----|
| B | 90  | E | 80  | C | 134 | A | 112 | D | 92 |
| E | 85  | D | 84  | B | 70  | C | 141 | A | 82 |
| C | 110 | A | 90  | D | 87  | B | 84  | E | 69 |
| A | 81  | C | 125 | E | 85  | D | 76  | B | 72 |
| D | 82  | B | 60  | A | 94  | E | 85  | C | 88 |

Sol:-

origin = subtract 100.

|   |     |   |     |   |     |   |     |   |     |
|---|-----|---|-----|---|-----|---|-----|---|-----|
| B | -10 | E | -20 | C | 34  | A | 12  | D | -8  |
| E | -15 | D | -16 | B | -30 | C | 41  | A | -18 |
| C | 10  | A | -10 | D | -13 | B | -16 | E | -31 |
| A | -19 | C | 25  | E | -15 | D | -24 | B | -28 |
| D | -18 | B | -40 | A | -6  | E | -15 | C | -12 |

$H_0$ : There's no significant difference b/w column means, row means and treatments.

$H_1$ : There is significant difference b/w column means, row means and treatments.

|       | $X_1$              | $X_2$              | $X_3$              | $X_4$             | $X_5$              | Total              | $X_1^2$               | $X_2^2$               | $X_3^2$               | $X_4^2$               | $X_5^2$               |
|-------|--------------------|--------------------|--------------------|-------------------|--------------------|--------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $Y_1$ | -10                | -20                | 34                 | 12                | -8                 | $\Sigma Y_1 = 8$   | 100                   | 400                   | 1156                  | 144                   | 64                    |
| $Y_2$ | -15                | -16                | -30                | 41                | -18                | $\Sigma Y_2 = -38$ | 225                   | 256                   | 900                   | 1681                  | 324                   |
| $Y_3$ | 10                 | -10                | -13                | -16               | -31                | $\Sigma Y_3 = -60$ | 100                   | 100                   | 169                   | 256                   | 961                   |
| $Y_4$ | -19                | 25                 | -15                | -24               | -28                | $\Sigma Y_4 = -61$ | 361                   | 625                   | 225                   | 576                   | 784                   |
| $Y_5$ | -18                | -40                | -6                 | -15               | -12                | $\Sigma Y_5 = -91$ | 324                   | 1600                  | 36                    | 225                   | 144                   |
| Total | $\Sigma X_1 = -52$ | $\Sigma X_2 = -61$ | $\Sigma X_3 = -30$ | $\Sigma X_4 = -2$ | $\Sigma X_5 = -97$ | $T = -242$         | $\Sigma X_1^2 = 1110$ | $\Sigma X_2^2 = 2981$ | $\Sigma X_3^2 = 2486$ | $\Sigma X_4^2 = 2882$ | $\Sigma X_5^2 = 2277$ |

step 1:  $N = 25$

step 2:  $T = -242$

step 3:  $\frac{T^2}{N} = \frac{(-242)^2}{25} = \frac{58564}{25} = 2342.56$

step 4:

$$TSS = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \Sigma X_4^2 + \Sigma X_5^2 - \frac{T^2}{N}$$

$$= 1110 + 2981 + 2486 + 2882 + 2277 - 2342.5$$

$$TSS = 9393.44$$

step 5:  $SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} + \frac{(\Sigma X_5)^2}{N_1} - \frac{T^2}{N}$

$$= \frac{(-52)^2}{5} + \frac{(-61)^2}{5} + \frac{(-30)^2}{5} + \frac{(-2)^2}{5} + \frac{(-97)^2}{5} - 2342.56$$

$$= 540.8 + 744.2 + 180 + 0.8 + 1881.8 - 2342.56$$

$$SSC = 1005.04$$

step 6:

$$SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} + \frac{(\Sigma Y_5)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{8^2}{5} + \frac{(-38)^2}{5} + \frac{(-60)^2}{5} + \frac{(-61)^2}{5} + \frac{(-91)^2}{5} - 2342.56$$

$$= 12.8 + 288.8 + 720 + 744.2 + 1656.2 - 2342.56$$

$$SSR = 1079.44$$

Step 7: To find SSK.

|   |     |     |     |     |       |                |
|---|-----|-----|-----|-----|-------|----------------|
| A | 12  | -18 | -10 | -19 | -16-6 | Total =<br>-41 |
| B | -10 | -30 | -16 | -28 | -40   | -124           |
| C | 34  | 41  | 10  | 25  | -12   | 98             |
| D | -8  | -16 | -13 | -24 | -18   | -79            |
| E | -20 | -15 | -31 | -15 | -15   | -96            |

$$SSK = \frac{(\sum A)^2}{N_3} + \frac{(\sum B)^2}{N_3} + \frac{(\sum C)^2}{N_3} + \frac{(\sum D)^2}{N_3} + \frac{(\sum E)^2}{N_3} - \frac{T^2}{N}$$

$$= \frac{(-41)^2}{5} + \frac{(-124)^2}{5} + \frac{(98)^2}{5} + \frac{(-79)^2}{5} + \frac{(-96)^2}{5} - 2342.5$$

$$= 336.2 + 3075.2 + 1920.8 + 1248.2 + 1843.2 - 2342.56$$

$$SSK = 6081.04$$

Step 8:

$$SSE = TSS - SSC - SSR - SSK$$

$$= 9893.44 - 1005.04 - 1079.44 - 6081.04$$

$$SSE = 1227.92$$

Step 9: A NOVA Table:-

| Source of Variation | Sum of Squares | Dof                                    | Mean Squares   | Variance ratio  | Table Value             |
|---------------------|----------------|--|--|---|-------------------------|
| Between columns     | SSC = 1005.04  | $\frac{K-1}{k-1} = 5-1 = 4$            | $MSC = \frac{SSC}{k-1}$<br>$= \frac{1005.04}{4}$<br>$MSC = 251.26$             | $F_c = \frac{MSC}{MSE}$<br>$= \frac{251.26}{102.32}$<br>$F_c = 2.455$   | $F_c(4,12)$<br>$= 3.26$ |
| Between rows        | SSR = 1079.44  | $k-1 = 4$                              | $MSR = \frac{SSR}{k-1}$<br>$= \frac{1079.44}{3}$<br>$MSR = 269.86$             | $F_R = \frac{MSR}{MSE}$<br>$= \frac{269.86}{102.82}$<br>$F_R = 2.637$   | $F_R(4,12)$<br>$= 3.26$ |
| Between treatment   | SSK = 6081.04  | $k-1 = 4$                              | $MSK = \frac{SSK}{k-1}$<br>$= \frac{6081.04}{4}$<br>$MSK = 1520.26$            | $F_T = \frac{MSK}{MSE}$<br>$= \frac{1520.26}{102.32}$<br>$F_T = 14.857$ | $F_T(4,12)$<br>$= 3.26$ |
| Errors              | SSE = 1227.92  | $(k-1) \times (k-2) = 4 \times 3 = 12$ | $MSE = \frac{SSE}{k-1 \times k-2}$<br>$= \frac{1227.92}{12}$<br>$MSE = 102.32$ |   |                         |

Step 10: Conclusion.

$\therefore F_c \text{ cal} < F_c \text{ tab}$ , we accept  $H_0$   
 $F_R \text{ cal} < F_R \text{ tab}$ , we accept  $H_0$   
 $F_T \text{ cal} > F_T \text{ tab}$ , we reject  $H_0$ .

3. Analyze the following Latin square experiments. The letters A, B, C, D denote the treatments and the figures in brackets denote the observations.

| Column<br>row | 1 |    | 2  |    | 3  |    | 4  |    |
|---------------|---|----|----|----|----|----|----|----|
|               | 1 | A  | 12 | D  | 20 | C  | 16 | B  |
| 2             | D | 18 | A  | 14 | B  | 11 | C  | 14 |
| 3             | B | 12 | C  | 15 | D  | 19 | A  | 13 |
| 4             | C | 16 | B  | 11 | A  | 15 | D  | 20 |

Sol:-

origin  $\Rightarrow$  subtract = 12.

|       |   | $X_1$ |   | $X_2$ |   | $X_3$ |   | $X_4$ |
|-------|---|-------|---|-------|---|-------|---|-------|
| $Y_1$ | A | 0     | D | 8     | C | 4     | B | -2    |
| $Y_2$ | D | 6     | A | 2     | B | -1    | C | 2     |
| $Y_3$ | B | 0     | C | 3     | D | 7     | A | 1     |
| $Y_4$ | C | 4     | B | -1    | A | 3     | D | 8     |

|       | $X_1$           | $X_2$           | $X_3$           | $X_4$          | Total           | $X_1^2$           | $X_2^2$           | $X_3^2$           | $X_4^2$           |
|-------|-----------------|-----------------|-----------------|----------------|-----------------|-------------------|-------------------|-------------------|-------------------|
| $Y_1$ | 0               | 8               | 4               | -2             | $\sum y_1 = 10$ | 0                 | 64                | 16                | 4                 |
| $Y_2$ | 6               | 2               | -1              | 2              | $\sum y_2 = 9$  | 36                | 4                 | 1                 | 4                 |
| $Y_3$ | 0               | 3               | 7               | 1              | $\sum y_3 = 11$ | 0                 | 9                 | 49                | 1                 |
| $Y_4$ | 4               | -1              | 3               | 8              | $\sum y_4 = 14$ | 16                | 1                 | 9                 | 64                |
| Total | $\sum x_1 = 10$ | $\sum x_2 = 12$ | $\sum x_3 = 13$ | $\sum x_4 = 9$ | $T = 44$        | $\sum x_1^2 = 52$ | $\sum x_2^2 = 78$ | $\sum x_3^2 = 75$ | $\sum x_4^2 = 73$ |

$H_0$ : There's no significant difference b/w  
column means, row means and  
treatments means.

$H_1$ : There is significant difference b/w  
column means, row means and  
treatments.

Step 1:  $N = 16$

Step 2:  $T = 44$

Step 3:  $\frac{T^2}{N} = \frac{44^2}{16} = 121$

Step 4:

$$TSS = \sum x_1^2 + \sum x_2^2 + \sum x_3^2 + \sum x_4^2 - \frac{T^2}{N}$$

$$= 52 + 78 + 75 + 73 - 121$$

$$TSS = 157$$

Step 5:

$$SSC = \frac{(\sum x_1)^2}{N_1} + \frac{(\sum x_2)^2}{N_1} + \frac{(\sum x_3)^2}{N_1} + \frac{(\sum x_4)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{10^2}{4} + \frac{12^2}{4} + \frac{13^2}{4} + \frac{9^2}{4} - 121$$

$$SSC = 2.5$$

Step 6:

$$SSR = \frac{(\sum y_1)^2}{N_2} + \frac{(\sum y_2)^2}{N_2} + \frac{(\sum y_3)^2}{N_2} + \frac{(\sum y_4)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{10^2}{4} + \frac{9^2}{4} + \frac{11^2}{4} + \frac{14^2}{4} - 121$$

$$SSR = 3.5$$

Step 7: To find SSK.

Arrange the elements on the order of treatments.

|   |   |    |    |    |            |
|---|---|----|----|----|------------|
| A | 0 | 2  | 3  | 1  | Total<br>6 |
| B | 0 | -1 | -1 | -2 | -4         |
| C | 4 | 3  | 4  | 2  | 13         |
| D | 6 | 8  | 7  | 8  | 29         |

$$SSK = \frac{(\sum A)^2}{N_1} + \frac{(\sum B)^2}{N_2} + \frac{(\sum C)^2}{N_3} + \frac{(\sum D)^2}{N_4}$$

$$= \frac{6^2}{4} + \frac{(-4)^2}{4} + \frac{(13)^2}{4} + \frac{(29)^2}{4}$$

$$SSK = 144.5$$

Step 8:  $SSE = TSS - SSC - SSR - SSK$

$$= 157 - 2.5 - 3.5 - 144.5$$

$$SSE = 6.5$$

Step 9: ANOVA Table:-

| Source of Variation | Sum of Squares | DoF                   | Mean Squares  | Variance Ratio   | Table Value       |
|---------------------|----------------|-----------------------|---|--|-------------------|
| Between Columns     | SSC = 2.5      | k-1 = 4-1 = 3         | MSC = $\frac{SSC}{k-1} = \frac{2.5}{3}$<br>MSC = 0.83                             | $F_c = \frac{MSE}{MSC} = \frac{1.08}{0.83}$<br>$F_c = 1.301$     | $F_c(6,3) = 8.94$ |
| Between Rows        | SSR = 3.5      | k-1 = 3               | M <sub>SR</sub> = $\frac{SSR}{k-1} = \frac{3.5}{3}$<br>M <sub>SR</sub> = 1.167    | $F_R = \frac{M_{SR}}{MSE} = \frac{1.167}{1.08}$<br>$F_R = 1.08$  | $F_R(3,6) = 4.76$ |
| Between treatments  | SSK = 144.5    | k-1 = 3               | M <sub>SK</sub> = $\frac{SSK}{k-1} = \frac{144.5}{3}$<br>M <sub>SK</sub> = 48.167 | $F_T = \frac{M_{SK}}{MSE} = \frac{48.167}{1.08}$<br>$F_T = 44.6$ | $F_T(3,6) = 4.76$ |
| Error               | SSE = 6.5      | k-1 x k-2 = 3 x 2 = 6 | MSE = $\frac{SSE}{6} = \frac{6.5}{6}$<br>MSE = 1.08                               |  |                   |

Step 10: Conclusion:

$F_c \text{ cal} < F_c \text{ tab}$ , we accept  $H_0$

$F_R \text{ cal} < F_R \text{ tab}$ , we accept  $H_0$

$F_T \text{ cal} > F_T \text{ tab}$ , we reject  $H_0$